MINISTRY OF EDUCATION, SINGAPORE
in collaboration with
CAMBRIDGE ASSESSMENT INTERNATIONAL EDUCATION
General Certificate of Education Advanced Level
Higher 2

MATHEMATICS

9758/01

Paper 1

For examination from 2025

SPECIMEN PAPER

3 hours

Additional Materials: Printed Answer Booklet
List of Formulae and Results (MF27)

READ THESE INSTRUCTIONS FIRST

Answer **all** questions.
Write your answers on the Printed Answer Booklet. Follow the instructions on the front cover of the answer booklet.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are **not** allowed in a question, you must present the mathematical steps using mathematical notations and not calculator commands.
You must show all necessary working clearly.

The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **5** printed pages and **1** blank page.

Singapore Examinations and Assessment Board

Cambridge Assessment International Education

1 A circular ink-blot is expanding such that the rate of change of its diameter D with respect to time t is $0.3\,\mathrm{cm\,s^{-1}}$. Find the rate of change of both the circumference and the area of the circle with respect to t when the radius of the circle is $2\,\mathrm{cm}$. [4]

2 The curve C with equation $y = x^3$ is transformed onto the curve with equation $y = \mathrm{f}(x)$ by a translation of 2 units in the negative x-direction, followed by a stretch of factor $\frac{1}{4}$ parallel to the y-axis.

(a) Write down the equation of the new curve. [1]

(b) Sketch C and the curve with equation $y = \mathrm{f}(x)$ on the same diagram, stating the values of the coordinates of the points where $y = \mathrm{f}(x)$ crosses the x- and y-axes. Find the x-coordinate(s) of the point(s) where the two curves intersect, giving your answer(s) correct to 3 decimal places. [4]

3 **(a)** Sketch the curve with equation $y = x - \dfrac{12}{x}$, giving the exact coordinates of the point(s) where the curve crosses the axes and the equations of any asymptotes. [4]

(b) Hence, or otherwise, solve the inequality $x - \dfrac{12}{x} < 1$. [3]

4 A student drops a ball vertically onto a hard surface and measures the height reached by the ball after each successive bounce. She drops the ball from an initial height of $160\,\mathrm{cm}$ and she estimates that the height the ball reaches after each bounce is $\frac{7}{8}$ of the height reached by the previous bounce.

(a) Find the total distance that the ball has travelled when it reaches the highest point after the third bounce. [2]

(b) The ball is considered to have stopped bouncing when a bounce first results in the height the ball reaches being less than $0.01\,\mathrm{cm}$. Find how many bounces the ball has made and the total distance that the ball has travelled in this case. [6]

5 The curve C has equation $y = \dfrac{1}{x}(\ln x)^3$, where $x > 1$.

(a) Find the exact x-coordinate, $x = x_1$, of the turning point on C and determine its nature. [4]

(b) Using calculus, find the exact area of the region between C, the line $y = 0$ and the lines with equations $x = \mathrm{e}$ and $x = x_1$. [3]

6 **(a)** The non-zero vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are such that $\mathbf{a} \times \mathbf{b} = \mathbf{c} \times \mathbf{a}$. Given that $\mathbf{b} \neq -\mathbf{c}$, find a linear relationship between $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$. [3]

(b) The variable position vector $\mathbf{v}$ satisfies the equation $\mathbf{v} \times (\mathbf{i} - 3\mathbf{k}) = 2\mathbf{j}$.

(i) Find the set of position vectors $\mathbf{v}$. [3]

(ii) Describe the set of points represented by position vectors $\mathbf{v}$ geometrically. [2]

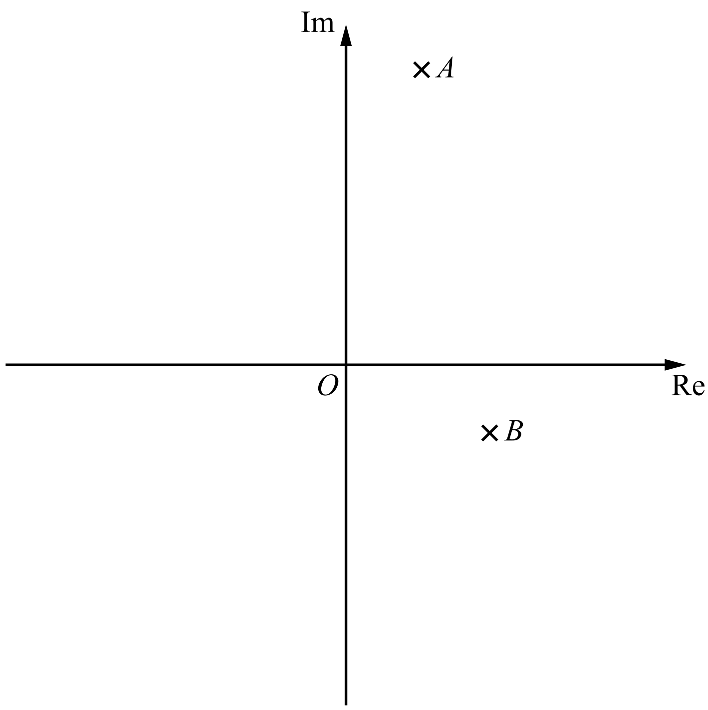
7 **Do not use a calculator in answering this question.**

(a) Showing your working, find the complex numbers z and w which satisfy the simultaneous equations.

$$2\mathrm{i}z + (1 - 2\mathrm{i})w = 4$$
$$(1 + \mathrm{i})z + (2 + \mathrm{i})w = 3$$

[6]

(b)



The points A and B on the Argand diagram represent the complex numbers w_1 and w_2 respectively.

(i) On the copy of the Argand diagram in the Printed Answer Booklet, plot the point C to represent the complex number w_3, where $w_3 = w_1 + w_2$. Show clearly the geometrical relationship between the points A, B and C. [2]

The points A', B' and C' represent the complex numbers $\mathrm{i}w_1$, $\mathrm{i}w_2$ and $\mathrm{i}w_3$ respectively.

(ii) On the same Argand diagram in the Printed Answer Booklet, plot the points A', B' and C'. State the transformation that maps the points A, B and C onto the points A', B' and C'. [2]

(iii) The transformation in **part (ii)** maps point A' onto the point A''. Determine, with justification, whether A'' represents the complex conjugate of w_1. [1]

8 The curve C has parametric equations

$$x = \cos^3 t, \qquad y = \sin^3 t, \qquad \text{where } 0 < t < \tfrac{1}{2}\pi.$$

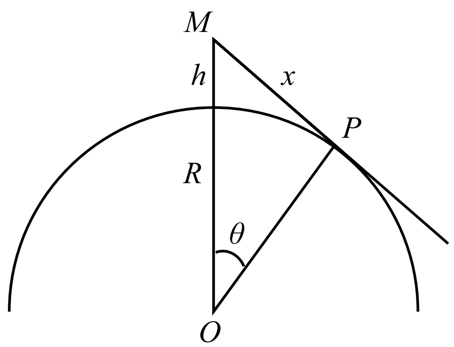
(a) Find the equation of the tangent to C at the point P with parameter p. [3]

(b) The tangent at P meets the x-axis at the point A and meets the y-axis at the point B. Find the length of AB. [3]

(c) Show that the cartesian equation of C may be written as $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 1$. [2]

(d) The region bounded by C, the line $x = 0$ and the line $y = 0$ is rotated through 2π about the y-axis. Find the volume of revolution of the solid formed, giving your answer correct to 3 decimal places. [4]

9



A man M is at the top of a mountain which is of height h km. The radius of the earth is assumed to be a constant R km. The furthest point on the earth's surface that the man can see is a point P such that $MP = x$ km and the angle $POM = \theta$, where O is the centre of the earth (see diagram). You may assume that the height of the man is negligible.

(a) Show that $x = (2hR)^{\frac{1}{2}}\left(1 + \dfrac{h}{2R}\right)^{\frac{1}{2}}$. [3]

(b) It is given that h is small compared to R. If $\alpha = \dfrac{h}{R}$, show that $\sin\theta \approx (2\alpha)^{\frac{1}{2}}\left(1 - \dfrac{3}{4}\alpha\right)$. [5]

(c) The man M has a scientific instrument which enables him to estimate the angle between PM and the horizontal. Given that this angle is $2°$ and that the radius of the earth is 6375 km, find estimates for the values of α and h. [4]

10 The point A has coordinates (0, 2, −3). The line l has equation $\frac{x}{2} = \frac{y+1}{-3} = \frac{z-2}{1}$.

(a) Find the cartesian equation of the plane π which contains A and is perpendicular to l. [3]

(b) Hence, or otherwise, find the coordinates of the point P on l which is closest to A. [3]

(c) The line m passes through the point with coordinates (4, –5, 10) and P. The line n lies in the same plane as l and m. Find a cartesian equation for n if n is the reflection of the line m the line l. [6]

11 The variable A is such that the rate of increase of A with respect to time t is proportional to the product of A and $(10 – A)$. The initial value of A is 2 and when $t = 5$ the value of A is 4.

(a) Write down a differential equation expressing the relation between A and t. Find the time at which $A = 8$, giving your answer correct to 2 decimal places. [8]

(b) Find the value of A when $t = 24$, giving your answer correct to 2 decimal places. [2]

(c) Write the solution of the differential equation in the form $A = \mathrm{f}(t)$ and sketch the graph of A against t. [4]

BLANK PAGE